AESB 2320 Pt 2 Re-exam 3 July 2015

1. The derivation is the same except that $q = C(T^4 - T_0^4)$, so

$$q_z|_z\, 2BW - q_z|_{z+\Delta z}\, 2BW - C(T^4 - T_0^4)\, 2W\Delta z = 0$$

Divide by $2BW\Delta z$; let $\Delta z \to 0$

$$-\frac{dq_z}{dz} - \frac{C}{B}(T^4 - T_0^4) = 0$$

Insert Fourier's law for q_z:

$$K\frac{d^2T}{dz^2} = \frac{C}{B}(T^4 - T_0^4)$$

2. a) We need h inside pipe to solve for U_0. $v = Q/(\pi R^2) = \frac{2\cdot 10^{-4}}{\pi(0.01)^2}$
$= 0.637\ m/s$

$$Re = (0.01)(0.637)(1000)/(0.001) = 6370$$

From chart*, (BSL Fig 14.3-2) $\frac{h_{ln} D}{K} Re^{-1} Pr^{-1/3} = 0.004$

$$\frac{h(0.02)}{0.68}\left(\frac{1}{12740}\right)\left(\frac{4190(0.001)}{(0.68)}\right)^{-1/3} = 0.004$$

$$h_{ln} = 3176 = \text{"}h_0\text{"}$$

$$U_0 = \frac{1}{0.01}\left[\frac{1}{(0.01)3176} + \frac{\ln(1.5/1)}{42.9} + \frac{1}{(0.015)25}\right]^{-1} \quad \text{(Eq. 9.6-31)}$$

$$= 100\left[\underbrace{0.031}_{\text{small effect}} + \underbrace{0.0095}_{\text{insignificant}} + \underbrace{2.67}_{\text{the big resistance}}\right]^{-1} = 36.9\ W/m^2K$$

b) Over 98% of the resistance is in convective heat transfer on the outside of the pipe. Increasing convection will increase h outside the pipe. This is the best approach. Reducing the other two resistances to <u>zero</u> would give less than 2% improvement.

* according to BSL, Eq. 14.3-16 is only for Re > 20,000. Some texts say this Eq. is OK for Re > 10,000, in which case it is OK here.

3. a) With the other sides insulated, this is like a slab w/ b=5 cm; the center of the slab is the bottom surface.

We want $(T-T_0)/(T_1-T_0) \leq (60-90)/(40-90) = 0.6$

$$\alpha = \frac{k}{\rho C_p} = \frac{0.6}{(1008)4180} = 1.42\cdot10^{-7}. \quad \text{At 10 min, } \frac{\alpha t}{b^2} = \frac{1.42\cdot10^{-7}(600)}{(0.05)^2} = 0.034$$

From Fig 12.1-1, this corresponds to $(y/b) \approx 0.86$, i.e. $\left(\frac{b-y}{b}\right) = 0.14$, measured from top.

The cooled layer is $\approx (0.14)(0.05) \approx 0.007$ m, or 7 mm thick.

b) There are two ways to bring the heat of the soup to the surface where it can be transferred to the surroundings: conduction + convection. They are "modes in parallel." If the soup were flowing, the convection could have only 1 effect: cool the soup off faster.

4. This is superposition. The first increase in T sets dim'less T. $T_0 = 50$ $T_1 = 150$. Dim'less T is $\frac{T-50}{150-50}$. The second T change is a increase of $\frac{200-150}{150-50} = 0.5$

$$\alpha = \frac{k}{\rho C_p} = \frac{386}{385.4(8890)} = 1.127\cdot10^{-4}$$

The geometry is a semi-infinite slab in the z direction. Because of the perfectly insulated surfaces, there is no conduction in the other two directions.

Effect of first T change: (5 min ago)
$$\frac{y}{\sqrt{4\alpha t}} = \frac{0.2}{\sqrt{4(1.27\cdot10^{-4})300}} = 0.51$$

from Fig 4.1-2, $\frac{T-T_0}{T_1-T_0} \approx 0.47$

Effect of 2nd change (3 min ago)
$$\frac{\alpha}{\sqrt{4\alpha t}} = \frac{0.2}{\sqrt{4(1.27\cdot10^{-4})180}} = 0.661$$

from Fig 4.1-2, $\frac{T-T_0}{T_1-T_0} \approx 0.35$

$$\frac{T-50}{150-50} = 0.51 + \left(\frac{1}{2}\right)(0.35) = 0.685$$

$$T = 118.5°C$$